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Static Shape Control of Space Trusses with Partial Measurements

Hiroshi Furuya*
Nagoya University, Chikusa, Nagoya 464-01, Japan
and
Raphael T. Haftka†
Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0203

The paper addresses the problem of predicting the statistics of shape distortion of space truss structures when measurements are limited to the distortions in a subset of the members of the truss. Expressions for the average values of a quadratic measure of the distortion in terms of mean values and covariances of member length errors are developed. An example of a 150-member antenna truss is used to assess the performance of static shape control with active members used as collocated sensors and actuators. Actuator locations are selected based on genetic algorithm optimization. It is found that performance is poor even when large numbers of sensor–actuator pairs are used. This indicates that it may not be practical to rely on incomplete measurements of member distortions to achieve reasonable levels of shape control.

Nomenclature

A	= weighting matrix defined by Eq. (4)
B	= positive semidefinite weighting matrix
$E(\cdot)$	= expected value
$E_m(\cdot)$	= conditional expected value (conditioned on $\epsilon_2 = \epsilon_{2m}$)
f	= mean squared error
f_c	= mean squared error after shape control with complete measurements
f_{cm}	= mean squared error after shape control with partial measurements
G	= coefficient matrix between displacement and member length error
g^2	= distortion ratio
L	= Factor of A defined by Eq. (6)
L_s	= Factor of S defined by Eq. (24)
n_a	= number of actuators
n_s	= population size in genetic algorithms
S	= weighting matrix defined by Eq. (24)
T	= linear control coefficient matrix
trace$(\cdot)$	= trace of matrix
u	= displacement field of truss structures
u_c	= corrective displacement field
$\Delta(\cdot)$	= uncorrelated value
$\Delta\epsilon$	= uncorrelated member length error
ϵ	= member length error vector
ϵ^p	= partially measured member length error vector
ϵ'	= normalized member length error defined by Eq. (5)
ϵ^0	= well-correlated member length error vector
ϵ_1	= unmeasured member length error vector [Eq. (10)]
ϵ_2	= measured member length error vector
θ	= vector of control commands
μ	= member error expected value vector, $E(\epsilon)$
μ_{1m}	= conditional expected value of unmeasured member length error with measured values
ρ	= correlation coefficient
$\Sigma(\cdot)$	= covariance matrix
$\Sigma_m(\cdot)$	= conditional covariance matrix of unmeasured member length error with measured values
Σ_0	= covariance matrix of member length error vector
$\sigma(\cdot)$	= standard deviation
$(\cdot)_m$	= conditional value with measured member length error
$(\cdot)^T$	= transposed matrix
$(\cdot)^0$	= well-correlated value
$(\cdot)^{-1}$	= inverse of matrix
$(\cdot)^*$	= result by random simulation

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*Assistant Professor, Department of Aerospace Engineering; currently Associate Professor, Department of Built Environment, Tokyo Institute of Technology, Yokohama 226, Japan. Member AIAA.

†Christopher Kraft Professor, Department of Aerospace and Ocean Engineering; currently Department of Aerospace Engineering Mechanics and Engineering Science, University of Florida, Gainesville, FL 32611. Associate Fellow AIAA.

Introduction

SPACE antennas often have to maintain extreme surface accuracies. This requirement translates to similar shape accuracy requirements for the truss structures that support these antennas. Member length errors are the prime contributors to shape distortion, and these length errors are mostly due to manufacturing errors and thermal expansion. Because of the random nature of both sources of length errors, the treatment of the shape distortion is often probabilistic in nature, with the expected value of some measure of surface distortion used as a measure of performance.[1]

To achieve the high accuracy, we usually use actuators that respond to shape distortion by corrective action. Most of the work on shape control of space trusses assumed perfect knowledge of the distortion. However, this assumption may not be realistic. Kuwao et al.[2] have explored shape estimation and control based on partial measurements. They found that when the distortions of some members are measured, interpolation of unmeasured member distortions may be an effective tool for estimating overall shape error and controlling it. Bruno et al.[3] have estimated the effects of incomplete measurements by a neural network approach. The objective of the present paper is to extend these works so as to include probabilistic treatment of the uncertainties associated with the distortion errors.

We assume that the major source of member length errors is thermal expansion. Manufacturing length errors are also important, but because they are constant, they may be compensated for electronically. That is, constant errors are easier to deal with than variable errors. Because temperature fields are continuous, we expect the temperatures of adjacent members to be correlated, and therefore their distortions to be likewise correlated.

This paper has two parts. In the first part we develop expressions for a quadratic shape distortion measure based on assumed knowledge of the statistics of the member length errors. We first consider the case of no actuators and the case of using actuators with full knowledge of the distortion. Next we consider the case of partial measurements and show that for this case the optimal control strategy starts by estimating the unmeasured quantities and then treats them the same as measured distortions. Finally, we obtain expressions for the loss of performance associated with partial measurements and use a simple example for illustration. The second part of the paper deals with the expected performance of shape correction based on partial measurements through an example of a 150-member truss antenna structure. We first propose a simple model for generating statistical parameters for such a structure, and then we consider the performance of shape correction as the number of sensors and actuators (assumed to be collocated) increases. To ensure that good locations are chosen for the actuators, we employ a genetic algorithm.[4]

Statistics of Shape Distortion

Shape Errors with Complete Measurements

Consider first the problem of estimating the shape errors when we know only the statistical distribution of the member length errors. The shape error is measured by a quadratic measure, such as the mean squared error

$$f = u^T B u \tag{1}$$

We assume a vector ϵ of member length errors is responsible for the displacement u, and the behavior is linear, so that u may be obtained from ϵ via linear analysis:

$$u = G\epsilon \tag{2}$$

where G is a matrix obtained (for example) by finite element analysis. Using Eqs. (1) and (2), we get

$$f = \epsilon^T A \epsilon \tag{3}$$

where

$$A = G^T B G \tag{4}$$

We assume that ϵ is a vector of normally distributed random variables with means given by the vector $\mu = E(\epsilon)$, and covariance given as $\Sigma_0 = \Sigma(\epsilon)$. To calculate the expected value of the error f, we start by defining

$$\epsilon' = L^T(\epsilon - \mu) \tag{5}$$

where L is any factor of A (e.g., the Cholesky factor) that satisfies

$$A = LL^T \tag{6}$$

Then ϵ' is a vector of normally distributed random variables with zero mean and covariance

$$\Sigma(\epsilon') = L^T \Sigma_0 L \tag{7}$$

With Eqs. (3), (5), and (6), we can write

$$f = (L^T\epsilon)^T L^T \epsilon = \epsilon'^T \epsilon' + 2\mu^T L \epsilon' + \mu^T A \mu \tag{8}$$

Finally, the expected value of f is given as

$$E(f) = \mathrm{trace}\left(L^T \Sigma_0 L\right) + \mu^T A \mu \tag{9}$$

Shape Errors with Partial Measurements

Next assume that we have measured some components of the vector ϵ. We partition the vector into its unmeasured part ϵ_1 and its measured part ϵ_2 , and denote the partially measured vector as ϵ^p:

$$\epsilon^p = \left[\epsilon_1^T, \epsilon_2^T\right]^T \tag{10}$$

We denote the expected values of the two parts as μ_1 and μ_2, respectively:

$$\mu_1 = E(\epsilon_1), \qquad \mu_2 = E(\epsilon_2) \tag{11}$$

We similarly partition the matrices A and Σ_0:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \qquad \Sigma_0 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \tag{12}$$

Using this notation, we have

$$f = \epsilon_1^T A_{11}\epsilon_1 + 2\epsilon_1^T A_{12}\epsilon_{2m} + \epsilon_{2m}^T A_{22}\epsilon_{2m} \tag{13}$$

We denote the conditional expected values and variances, given $\epsilon_2 = \epsilon_{2m}$, by subscript m. Then, from Eq. (13),

$$E_m(f) = E(f \mid \epsilon_2 = \epsilon_{2m})$$

$$= E_m(f_1) + 2\mu_{1m}^T A_{12}\epsilon_{2m} + \epsilon_{2m}^T A_{22}\epsilon_{2m} \tag{14}$$

where

$$f_1 = \epsilon_1^T A_{11}\epsilon_1 \tag{15}$$

The conditional expected value μ_{1m} and the conditional covariance matrix Σ_{11m} of ϵ_1 are given (e.g., Ref. 5; see also Appendix A) as

$$\mu_{1m} = E_m(\epsilon_1) = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(\epsilon_{2m} - \mu_2) \tag{16}$$

and

$$\Sigma_{11m} = \Sigma_m(\epsilon_1) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T \tag{17}$$

To evaluate $E_m(f_1)$ we can use the results of the previous section with ϵ_1 replacing ϵ, A_{11} replacing A, and the conditional expected values and covariances replacing their unconditional counterparts. Finally we obtain the following expression:

$$E_m(f) = \mathrm{trace}\left(L_{11}^T \Sigma_{11m} L_{11}\right) + \mu_{1m}^T A_{11}\mu_{1m}$$

$$+ 2\mu_{1m}^T A_{12}\epsilon_{2m} + \epsilon_{2m}^T A_{22}\epsilon_{2m} \tag{18}$$

where L_{11} is a factor of A_{11}.

Shape Correction with Complete Measurements

We assume that we have a vector θ of control commands that produce a vector of corrective displacements u_c that depends linearly on the control:

$$u_c = T\theta \tag{19}$$

The total displacement field u_T is given as

$$u_T = G\epsilon + T\theta \tag{20}$$

Assuming that we have complete knowledge of the effect of the errors, that is, we know $G\epsilon$, we can calculate the best corrective action θ by minimizing f:

$$f = (G\epsilon + T\theta)^T B (G\epsilon + T\theta) \tag{21}$$

Differentiating f with respect to θ and setting to zero, we find that

$$\theta = -(T^T B T)^{-1} T^T B G \epsilon \tag{22}$$

Note that when the columns of BT are linearly dependent, or when one of the columns is zero (which happens when an actuator does not have any effect on any of the components of u that have nonzero B entries), then $T^T BT$ is singular. In this case we understand the inverse to mean the generalized inverse (which we have implemented using the singular-value decomposition). Substituting θ into Eq. (21), we get the corrected value of f,

$$f_c = \epsilon^T (A - S)\epsilon \tag{23}$$

where

$$S = G^T B T (T^T B T)^{-1} T^T B G = L_s L_s^T \tag{24}$$

We can calculate the expected value of $\epsilon^T S\epsilon$ using a procedure similar to the one that led to Eq. (9) with A replaced by S and ϵ' defined on the basis of the Cholesky factor L_s of S instead of the Cholesky factor L of A. Altogether we get

$$E(f_c) = \mathrm{trace}\left(L^T \Sigma_0 L\right) + \mu^T A\mu - \mathrm{trace}\left(L_s^T \Sigma_0 L_s\right) - \mu^T S\mu \quad (25)$$

This expression can be further minimized by choosing optimal locations of sensors and actuators.

Shape Correction with Partial Measurements

Next we consider the problem of applying corrections on the basis of incomplete data. We assume that we select θ so as to minimize the conditional expected value of f. The conditional expected value of f with respect to the measured strain from Eq. (21) and with Eq. (4) yields

$$E_m(f) = E_m(\epsilon^{pT} A\epsilon^p) + 2E_m(\epsilon^p)^T G^T BT^T\theta + \theta^T T^T BT\theta \quad (26)$$

where

$$E_m(\epsilon^p) = \left[\mu_{1m}^T,\ \epsilon_{2m}^T\right]^T \quad (27)$$

Differentiating with respect to θ and setting to zero, we get

$$\theta = -(T^T BT)^{-1} T^T BGE_m(\epsilon^p) \quad (28)$$

Equation (28) is similar to Eq. (22) and indicates that in calculating the optimal actuator action we use the expected value of the strains for unmeasured elements. This means that with partial measurements of the strains, we will first estimate the unmeasured strains from Eq. (16) and then use these strains to calculate the actuator response as if these strains had been measured. We then obtain the corrected value of f by substituting from Eq. (28) back into Eq. (21) to get

$$f_{cm} = \epsilon^{pT} A\epsilon^p - \epsilon^{pT} SE_m(\epsilon^p) - E_m(\epsilon^p)^T S\epsilon^p + E_m(\epsilon^p)^T SE_m(\epsilon^p) \quad (29)$$

The expected value of f_{cm} may be obtained with some algebraic manipulations (see Appendix B) as

$$E(f_{cm}) = E(f_c) + \mathrm{trace}\left(L_s^T \begin{bmatrix} \Sigma_{11m} & 0 \\ 0 & 0 \end{bmatrix} L_s\right) \quad (30)$$

The difference between the expected value with complete and partial measurement will depend on the number of sensors and their locations.

Two-Bar Truss Example

Consider the two-bar truss structure in Fig. 1. The relationships between the member length errors and the displacements are

$$(l + \epsilon_1)^2 = \left(\tfrac{1}{2}\sqrt{3}l + u_1\right)^2 + \left(\tfrac{1}{2}l + u_2\right)^2 \quad (31)$$

$$\left(\tfrac{1}{2}l + \epsilon_2\right)^2 = u_1^2 + \left(\tfrac{1}{2}l + u_2\right)^2 \quad (32)$$

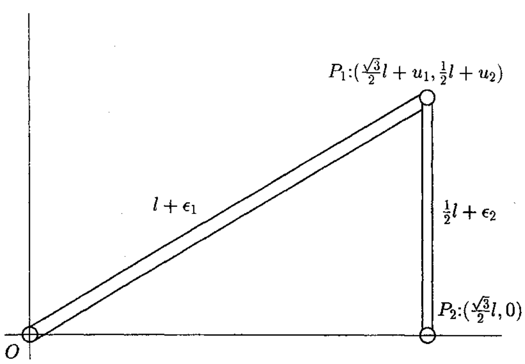

Fig. 1 Two-bar truss structure model.

By linearizing we get

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \end{Bmatrix} = \begin{bmatrix} \frac{1}{2}\sqrt{3} & \frac{1}{2} \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = G^{-1}u \quad (33)$$

where

$$G = \begin{bmatrix} \frac{2}{3}\sqrt{3} & -\frac{1}{3}\sqrt{3} \\ 0 & 1 \end{bmatrix} \quad (34)$$

We take the weighting matrix B as the unit matrix

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (35)$$

and then

$$L = G^T \quad (36)$$

and

$$A = G^T G = \begin{bmatrix} \frac{4}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{4}{3} \end{bmatrix} \quad (37)$$

The expected values and correlation matrix of the member length errors are

$$\mu = [\mu_1, \mu_2]^T \quad (38)$$

and

$$\Sigma_0 = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \quad (39)$$

Also

$$\mu^T A\mu = \tfrac{4}{3}\left(\mu_1^2 - \mu_1\mu_2 + \mu_2^2\right) \quad (40)$$

and, from Eq. (7),

$$\Sigma(\epsilon') = \begin{bmatrix} \frac{1}{3}\left(4\sigma_1^2 - 4\rho\sigma_1\sigma_2 + \sigma_2^2\right) & \frac{1}{3}\sqrt{3}\left(2\rho\sigma_1\sigma_2 - \sigma_2^2\right) \\ \frac{1}{3}\sqrt{3}\left(2\rho\sigma_1\sigma_2 - \sigma_2^2\right) & \sigma_2^2 \end{bmatrix} \quad (41)$$

Thus, from Eq. (9), the expected value of f without measurement is

$$F(f) = \tfrac{4}{3}\left(\sigma_1^2 - \rho\sigma_1\sigma_2 + \sigma_2^2 + \mu_1^2 - \mu_1\mu_2 + \mu_2^2\right) \quad (42)$$

We assume element 2 is an active member, so that T is the second column of G,

$$T = \left(-\tfrac{1}{3}\sqrt{3}, 1\right)^T \quad (43)$$

Thus,

$$S = G^T BT(T^T BT)^{-1} T^T BG = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{4}{3} \end{bmatrix} \quad (44)$$

and a possible factor L_s of S is

$$L_s = \left(-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)^T \quad (45)$$

If we put the sensor on element 2, the conditional covariance matrix is

$$\Sigma_{11m} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T = (1 - \rho^2)\sigma_1^2$$

Next we calculate the following expected value:

$$E(\epsilon^T S\epsilon) = \mathrm{trace}\left(L_s^T \Sigma_0 L_s\right) + \mu^T S\mu \quad (46)$$

where

$$L_s^T \Sigma_0 L_s = \tfrac{1}{3}\left(\sigma_1^2 - 4\rho\sigma_1\sigma_2 + 4\sigma_2^2\right) \quad (47)$$

and

$$\mu^T S\mu = \tfrac{1}{3}\left(\mu_1^2 - 4\mu_1\mu_2 + 4\mu_2^2\right) \quad (48)$$

Using these values, we finally obtain the expected values with complete measurements and with partial measurements, respectively, from Eq. (25),

$$E(f_c) = \sigma_1^2 + \mu_1^2 \tag{49}$$

and from Eq. (30),

$$E(f_{cm}) = \sigma_1^2 + \mu_1^2 + \tfrac{1}{3}(1-\rho^2)\sigma_1^2 \tag{50}$$

If instead of using element 2 as the active element we use element 1, we get the expected symmetrical results

$$E(f_c) = \sigma_2^2 + \mu_2^2 \tag{51}$$

and

$$E(f_{cm}) = \sigma_2^2 + \mu_2^2 + \tfrac{1}{3}(1-\rho^2)\sigma_2^2 \tag{52}$$

Comparing these results with $E(f)$, we note that besides eliminating the effect of the error in the active member (i.e., no contribution from σ_2 or μ_2 for the case that element 2 is the active element), we are able to reduce the effect of the error in the other member as well. With complete measurements this reduction is by one-quarter (from $\frac{4}{3}$ to 1). With partial measurement there is an extra penalty, unless the correlation coefficient ρ is equal to 1. Indeed, when the correlation is perfect, knowledge of one component implies knowledge of the other one as well.

Performance of Shape Correction with Partial Measurements

The algebraic expressions developed in the previous section do not tell us how well we can expect to correct shape distortion with partial measurements. To get an answer to this question we turn our attention to a more realistic example than the one analyzed in the previous section, the 150-member truss shown in Fig. 2. To make use of this example we need to come up with some reasonable statistics for member length errors and a strategy for optimally placing actuators on the truss.

Simple Statistical Model

To generate statistical data for our problem we assume that the vector of length errors ϵ is composed of two uncorrelated contributions

$$\epsilon = \epsilon^0 + \Delta\epsilon \tag{53}$$

where ϵ^0 represents the well-correlated part of the distortion field, due to continuous thermal fields, and $\Delta\epsilon$ represents random, uncorrelated fluctuations. We denote the mean and covariance matrix of ϵ^0 by and μ^0 and Σ^0, respectively, are the corresponding quantities for $\Delta\epsilon$ as $\Delta\mu$ and $\Delta\Sigma$, respectively. Here we assume that $\Delta\mu = 0$, and that the correlation between all the components of ϵ^0 is one, so that Σ^0 is completely defined in terms of the standard deviations σ_i of its components:

$$\Sigma_{ij}^0 = \sigma_i\sigma_j \tag{54}$$

The mean μ and covariance Σ of ϵ are then given as

$$\mu = E(\epsilon) = E(\epsilon^0 + \Delta\epsilon) = \mu^0 \tag{55}$$

$$\begin{aligned}\Sigma &= E[(\epsilon-\mu)(\epsilon-\mu)^T]\\ &= E[(\epsilon^0-\mu^0)(\epsilon^0-\mu^0)^T] + E[(\Delta\epsilon-\Delta\mu)(\Delta\epsilon-\Delta\mu)^T]\\ &= \Sigma^0 + \Delta\Sigma\end{aligned} \tag{56}$$

where we have used the assumption that ϵ^0 and $\Delta\epsilon$ are uncorrelated.

With this simple model of member length errors, the relative magnitude of the standard deviations of ϵ^0 and $\Delta\epsilon$ determine the correlation between the components of ϵ, as illustrated by the following two-member example.

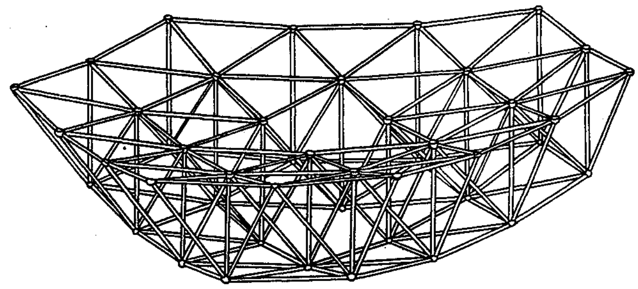

Fig. 2 JPL truss antenna model.

Consider a case where member 1 and member 2 are subjected to thermal loads with the temperature differential (from some ground state) for member 2 expected to be twice as large as the differential for member 1. This would lead to length errors of $(\epsilon_1, 2\epsilon_1)$, with a covariance matrix of

$$\Sigma^0 = \begin{bmatrix}\sigma_1^2 & 2\sigma_1^2\\ 2\sigma_1^2 & 4\sigma_1^2\end{bmatrix} \tag{57}$$

We also have some uncorrelated noise for both members, and if it is of the same magnitude for both, then

$$\Delta\Sigma = \begin{bmatrix}\Delta\sigma^2 & 0\\ 0 & \Delta\sigma^2\end{bmatrix} \tag{58}$$

and

$$\Sigma = \Sigma^0 + \Delta\Sigma = \begin{bmatrix}\sigma_1^2 + \Delta\sigma^2 & 2\sigma_1^2\\ 2\sigma_1^2 & 4\sigma_1^2 + \Delta\sigma^2\end{bmatrix} \tag{59}$$

The correlation coefficient ρ between ϵ_1 and ϵ_2 is

$$\rho = \frac{\Sigma_{12}}{\sqrt{\Sigma_{11}\Sigma_{22}}} = \frac{2\sigma_1^2}{\sqrt{(\sigma_1^2+\Delta\sigma^2)(4\sigma_1^2+\Delta\sigma^2)}} \tag{60}$$

Note that if $\Delta\epsilon$ is small compared to ϵ_1, then $\Delta\sigma$ will be small compared to σ, and then the correlation coefficient will be close to 1. If, on the other hand, $\Delta\epsilon$ dominates, $\Delta\sigma$ will be large and the correlation coefficient will be close to zero. Thus, if we have some idea about the fraction of ϵ_1 that can be determined from interpolation and the part that is random, we can estimate σ and $\Delta\sigma$, and we can calculate the covariance matrix of these two member length errors.

The expressions developed in this paper for the expected value of the shape distortion do not depend on the details of the statistical distributions, but only on their mean values and covariances. However, for the examples we have also performed simulations by randomly generating shape errors. These simulations employed normally distributed member error vectors.

Selection of Actuator Locations by Genetic Algorithm

The locations of actuators are selected so as to minimize the expected value of the shape distortion. We furthermore assume that active members are used as both actuators and sensors, so that the actuators and sensors are collocated. Using Eqs. (25) and (30), we get for the expected value of the distortion

$$E(f_{cm}) = \text{trace}\left(L^T\Sigma_0 L\right) + \mu^T A\mu - \text{trace}\left(L_s^T\Sigma_0 L_s\right) - \mu^T S\mu + \text{trace}\left(L_s^T\begin{bmatrix}\Sigma_{11m} & 0\\ 0 & 0\end{bmatrix}L_s\right) \tag{61}$$

As the first and second terms of the right-hand side of the equation are independent of the location of actuators, the problem reduces to the maximization of

$$\text{trace}\left(L_s^T\Sigma_0 L_s\right) + \mu^T S\mu - \text{trace}\left(L_s^T\begin{bmatrix}\Sigma_{11m} & 0\\ 0 & 0\end{bmatrix}L_s\right) \tag{62}$$

Table 1 Distortion ratios with complete and partial measurement: analytical results and random simulations for reflector surface z displacements

a) $\rho = 0.99999$, $\mu = 0.001$, $\sigma = 0.001$

Number of actuators	With complete measurement: Analytical value $E(f_c)/E(f)$	With complete measurement: Random simulation $E(f_c^*)/E(f^*)$	With complete measurement: Random simulation $\sigma(f_c^*)/E(f^*)$	With partial measurement: Analytical value $E(f_{cm})/E(f)$	With partial measurement: Random simulation $E(f_{cm}^*)/E(f^*)$	With partial measurement: Random simulation $\sigma(f_{cm}^*)/E(f^*)$
2	0.4653	0.4653	0.5418	0.4653	0.4653	0.5418
4	0.0604	0.0604	0.0708	0.0605	0.0605	0.0708
6	0.0338	0.0338	0.0421	0.0338	0.0339	0.0421
8	0.0189	0.0189	0.0231	0.0189	0.0189	0.0231
16	0.0017	0.0017	0.0021	0.0018	0.0018	0.0021
24	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
32	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000
40	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000

$E(f) = 0.1794 \times 10^{-4}$, $E(f^*) = 0.1827 \times 10^{-4}$, $\sigma(f^*) = 0.2213 \times 10^{-4}$

b) $\rho = 0.5$, $\mu = 0.001$, $\rho = 0.001$

Number of actuators	With complete measurement: Analytical value $E(f_c)/E(f)$	With complete measurement: Random simulation $E(f_c^*)/E(f^*)$	With complete measurement: Random simulation $\sigma(f_c^*)/E(f^*)$	With partial measurement: Analytical value $E(f_{cm})/E(f)$	With partial measurement: Random simulation $E(f_{cm}^*)/E(f^*)$	With partial measurement: Random simulation $\sigma(f_{cm}^*)/E(f^*)$
2	0.5957	0.5918	0.3367	0.8854	0.8741	0.5032
4	0.3265	0.3317	0.1426	0.7523	0.7364	0.4417
6	0.2828	0.2797	0.1275	0.6695	0.6745	0.3837
8	0.2420	0.2497	0.1155	0.6000	0.6239	0.3450
16	0.0881	0.0886	0.0453	0.4916	0.4953	0.2511
24	0.0179	0.0179	0.0162	0.4433	0.4416	0.2245
32	0.0000	0.0000	0.0000	0.3980	0.4092	0.2160
40	0.0000	0.0000	0.0000	0.3834	0.3878	0.2455

$E(f) = 0.1325 \times 10^{-3}$, $E(f^*) = 0.1303 \times 10^{-3}$, $\sigma(f^*) = 0.7897 \times 10^{-4}$

c) $\rho = 0.0$, $\mu = 0.001$, $\sigma = 0.001$

Number of actuators	With complete measurement: Analytical value $E(f_c)/E(f)$	With complete measurement: Random simulation $E(f_c^*)/E(f^*)$	With complete measurement: Random simulation $\sigma(f_c^*)/E(f^*)$	With partial measurement: Analytical value $E(f_{cm})/E(f)$	With partial measurement: Random simulation $E(f_{cm}^*)/E(f^*)$	With partial measurement: Random simulation $\sigma(f_{cm}^*)/E(f^*)$
2	0.5891	0.5936	0.3787	0.8998	0.8881	0.5528
4	0.3427	0.3458	0.1446	0.7899	0.8001	0.4806
6	0.2998	0.3000	0.1306	0.7132	0.6992	0.3692
8	0.2577	0.2591	0.1218	0.6376	0.6352	0.3299
16	0.0791	0.0800	0.0443	0.5229	0.5283	0.2595
24	0.0130	0.0124	0.0104	0.5131	0.5085	0.2840
32	0.0000	0.0000	0.0000	0.4758	0.4783	0.2585
40	0.0000	0.0000	0.0000	0.3998	0.4048	0.1957

$E(f) = 0.2471 \times 10^{-3}$, $E(f^*) = 0.2459 \times 10^{-3}$, $\sigma(f^*) = 0.1525 \times 10^{-3}$

The genetic algorithm used for obtaining actuator locations is an integer-coding uniform crossover algorithm[4] described briefly in the following. For n_a actuators the locations are coded as a string of n_a integers specifying the active members. Genetic algorithms work simultaneously on a population of n_s designs (strings), subjecting them to operators that mimic the biological processes of evolution and natural selection. The procedure goes through the following steps:

1) Generate randomly an initial population of n_s designs.

2) Evaluate the objective function for each member of the population, and rank the members.

3) Select a pair of parents from the population by simulating a roulette wheel, with each design getting a portion of the wheel proportional to n_s minus its rank.

4) Generate a child design by uniform crossover. That is, for each string position take a location from one parent or the other parent at random.

5) Mutate randomly (but with low probability) some of the actuator locations in the child design to random new locations.

6) Repeat steps 2–5 n_s times to generate a complete new generation.

7) Replace the old generation with the new generation.

This process is repeated for a specified number of generations.

JPL Truss Antenna Model

The Jet Propulsion Laboratory (JPL) truss antenna model shown in Fig. 2 was used to evaluate the performance of shape correction based on partial measurements. The truss has 150 members and 45 nodes. Of these nodes, 27 support the reflecting surface, and we assumed that only the z displacement (axisymmetric direction of the parabola) of these nodes affects shape distortion. Accordingly, we used a diagonal matrix B with zeros everywhere expect for the 27 locations corresponding to the z displacements of the nodes of the reflecting surface.

For testing performance, we have assumed uniform heating of the entire truss. Thus the perfectly correlated part of the member errors has all members with the same mean value and standard deviation of the error. The uncorrelated part of the error has zero mean with all

Table 2 Distortion-correction ratios with complete and partial measurement: random simulations for reflector surface z displacements

a) $\rho = 0.99999$, $\mu = 0.001$, $\sigma = 0.001$

Number of actuators	Distortion-correction ratio: With complete measurement		With partial measurement	
	$E(f_c^*/f^*)$	$\sigma(f_c^*/f^*)$	$E(f_{cm}^*/f^*)$	$\sigma(f_{cm}^*/f^*)$
2	0.4659	0.0239	0.4691	0.0305
4	0.0629	0.0183	0.0671	0.0451
6	0.0376	0.0289	0.0417	0.0523
8	0.0220	0.0221	0.0284	0.0612
16	0.0030	0.0081	0.0129	0.0786
24	0.0002	0.0020	0.0089	0.0587
32	0.0000	0.0000	0.0068	0.0537
40	0.0000	0.0000	0.0058	0.0381

$E(f^*) = 0.1827 \times 10^{-4}, \sigma(f^*) = 0.2213 \times 10^{-4}$

b) $\rho = 0.5$, $\mu = 0.001$, $\sigma = 0.001$

Number of actuators	Distortion-correction ratio: With complete measurement		With partial measurement	
	$E(f_c^*/f^*)$	$\sigma(f_c^*/f^*)$	$E(f_{cm}^*/f^*)$	$\sigma(f_{cm}^*/f^*)$
2	0.6516	0.2170	0.9202	0.2624
4	0.4035	0.1925	0.8112	0.3461
6	0.3528	0.1904	0.7813	0.4089
8	0.3119	0.1707	0.7305	0.3920
16	0.1167	0.0843	0.6085	0.3490
24	0.0233	0.0250	0.5537	0.3668
32	0.0000	0.0000	0.5135	0.3457
40	0.0000	0.0000	0.4940	0.3283

$E(f^*) = 0.1303 \times 10^{-3}, \sigma(f^*) = 0.7897 \times 10^{-4}$

c) $\rho = 0.0$, $\mu = 0.001$, $\sigma = 0.001$

Number of actuators	Distortion-correction ratio: With complete measurement		With partial measurement	
	$E(f_c^*/f^*)$	$\sigma(f_c^*/f^*)$	$E(f_{cm}^*/f^*)$	$\sigma(f_{cm}^*/f^*)$
2	0.6485	0.2169	0.9334	0.2744
4	0.4272	0.2018	0.8902	0.4004
6	0.3786	0.2007	0.8090	0.3964
8	0.3248	0.1747	0.7545	0.3994
16	0.1052	0.0773	0.6474	0.3811
24	0.0169	0.0186	0.6260	0.4008
32	0.0000	0.0000	0.5912	0.3675
40	0.0000	0.0000	0.5146	0.3213

$E(f^*) = 0.2459 \times 10^{-3}, \sigma(f^*) = 0.1525 \times 10^{-3}$

members having the same standard deviations. We considered three ratios between the variance (square of the standard deviation) of the correlated part and the total variance (the sum of the two parts): zero, one-half, and one. These assumptions also make for a uniform coefficient of correlation between all members equal to the ratio of the correlated variance and total variance. This case appears to be fairly benign in terms of the distribution of errors, but, as the results show, it is not easy to control the shape distortion.

Reflector Surface Distortion Results

The effectiveness of the shape correction is measured in terms of ratios of corrected shape distortion to original shape distortions. From the expressions derived in the paper we can calculate the ratio g^2 of the means of the two quantities

$$g^2 = \frac{E(f_{cm})}{E(f)} \tag{63}$$

which is called the distortion ratio. From Monte Carlo simulations we can calculate besides g^2 also several other quantities:

$$E\left(\frac{f_{cm}}{f}\right), \quad \frac{\sigma(f_{cm})}{E(f)}, \quad \sigma\left(\frac{f_{cm}}{f}\right) \tag{64}$$

The first is the average of the distortion-correction ratio, the second is a normalized standard deviation of corrected distortion, and the third is the standard deviation of the distortion-correction ratio.

Distortion ratios with complete and partial measurements were obtained by the analytical formulas presented in the paper and by 1000 random (Monte Carlo) simulations. Results for surface error of the truss reflector antenna are shown in Table 1. In the table, an asterisk indicates the results by random simulation. The agreement between the analytical and simulation results is very good. It may appear that with the possibility of performing such simulations there is no need for analytical results. However, the genetic optimization of the actuator locations would have been prohibitively expensive if we had needed to rely on simulations instead of analytical expressions.

As can be seen from Table 1, when the correlation coefficient is very close to one, the distortion ratios with complete and partial measurement are very close. This is due to the fact that with all members highly correlated, the partial measurements provide excellent estimates of the unmeasured distortions. Table 1a also indicates that we can get excellent performance with a small number of actuators. For example, four actuators suffice to reduce the expected distortion to 6% of its original value. This excellent performance reflects the uniformity of the distortion field and the relatively small number (below 27) of displacement components that define the distortion measure.

Table 3 Distortion ratios with complete and partial measurement: analytical results and random simulations for all displacements

a) $\rho = 0.99999$, $\mu = 0.001$, $\sigma = 0.001$

	Distortion ratio					
	With complete measurement			With partial measurement		
	Analytical value	Random simulation		Analytical value	Random simulation	
Number of actuators	$E(f_c)/E(f)$	$E(f_c^*)/E(f^*)$	$\sigma(f_c^*)/E(f^*)$	$E(f_{cm})/E(f)$	$E(f_{cm}^*)/E(f^*)$	$\sigma(f_{cm}^*)/E(f^*)$
2	0.9285	0.9285	1.1902	0.9285	0.9285	1.1902
4	0.8395	0.8395	0.9716	0.8395	0.8395	0.9716
6	0.7538	0.7537	0.8588	0.7538	0.7538	0.8588
8	0.6888	0.6888	0.8255	0.6888	0.6888	0.8255
16	0.4694	0.4694	0.5680	0.4694	0.4694	0.5680
24	0.4134	0.4134	0.4850	0.4134	0.4134	0.4850
32	0.3393	0.3393	0.4023	0.3393	0.3393	0.4023
40	0.2798	0.2798	0.3284	0.2798	0.2798	0.3284

$E(f^*) = 0.3117 \times 10^{-3}$, $E(F^*) = 0.3128 \times 10^{-3}$, $\sigma(f^*) = 0.3727 \times 10^{-3}$

b) $\rho = 0.5$, $\mu = 0.001$, $\sigma = 0.001$

	Distortion ratio					
	With complete measurement			With partial measurement		
	Analytical value	Random simulation		Analytical value	Random simulation	
Number of actuators	$E(f_c)/E(f)$	$E(f_c^*)/E(f^*)$	$\sigma(f_c^*)/E(f^*)$	$E(f_{cm})/E(f)$	$E(f_{cm}^*)/E(f^*)$	$\sigma(f_{cm}^*)/E(f^*)$
2	0.8031	0.8059	0.5064	0.9399	0.9380	0.5462
4	0.6807	0.6767	0.4896	0.8785	0.8724	0.5412
6	0.5987	0.5980	0.3930	0.8163	0.8167	0.4591
8	0.4959	0.5006	0.3668	0.7466	0.7541	0.4175
16	0.3874	0.3952	0.3026	0.5907	0.5942	0.3241
24	0.2665	0.2677	0.1883	0.5240	0.5231	0.2528
32	0.2108	0.2116	0.1492	0.4168	0.4200	0.1868
40	0.1717	0.1716	0.1283	0.3791	0.3831	0.1729

$E(f) = 0.5367 \times 10^{-3}$, $E(f^*) = 0.5340 \times 10^{-3}$, $\sigma(f^*) = 0.3055 \times 10^{-3}$

c) $\rho = 0.0$, $\mu = 0.001$, $\sigma = 0.001$

	Distortion ratio					
	With complete measurement			With partial measurement		
	Analytical value	Random simulation		Analytical value	Random simulation	
Number of actuators	$E(f_c)/E(f)$	$E(f_c^*)/E(f^*)$	$\sigma(f_c^*)/E(f^*)$	$E(f_{cm})/E(f)$	$E(f_{cm}^*)/E(f^*)$	$\sigma(f_{cm}^*)/E(f^*)$
2	0.7285	0.7187	0.2607	0.9213	0.9214	0.4008
4	0.5634	0.5677	0.1471	0.8419	0.8395	0.3096
6	0.4689	0.4655	0.1024	0.7641	0.7603	0.2759
8	0.4014	0.4056	0.0859	0.7202	0.7298	0.2662
16	0.2904	0.2897	0.0537	0.5841	0.5809	0.2030
24	0.2151	0.2163	0.0382	0.5123	0.5161	0.1835
32	0.1960	0.1968	0.0334	0.4580	0.4596	0.1406
40	0.1633	0.1646	0.0286	0.4178	0.4142	0.1340

$E(f) = 0.7618 \times 10^{-3}$, $E(f^*) = 0.7557 \times 10^{-3}$, $\sigma(f^*) = 0.3395 \times 10^{-3}$

As can be seen from Tables 1b and 1c, without such perfect correlation the number of actuators required to reduce the distortion is substantially larger even with complete measurements. For example, 16 actuators reduce the expected distortion by just over an order of magnitude, while for the perfect correlation case they reduce it by almost three orders of magnitude. Note that the numbers in the first column of Table 1b are slightly higher than the corresponding numbers in Table 1c, which gives the impression that the case with a correlation of 0.5 is more difficult than the case with almost zero correlation. However, $E(f)$ is almost twice as high in Table 1c, so that both the uncorrected and corrected distortions are higher for the low-correlation case.

The results with partial measurements are quite discouraging, with 40 pairs of sensors and actuators not being sufficient for reducing the expected value of the distortion by a factor of 3. The random simulation also gives the standard deviations associated with the distortion ratio. The large standard deviation reinforces the bleakness of the results for correlations of 0.001 and 0.5. These results indicate that when the errors have a substantial uncorrelated random component, partial measurement may not be a viable strategy.

Table 2 shows the distortion-correction ratio obtained by random simulations. The mean values of the distortion-correction ratio are somewhat larger than the corresponding values of the distortion ratio in Table 1. This difference may reflect the influence of cases where f is small but not easily correctible by the actuator configuration.

When the number of displacement components of interest is larger than the 27 we have for this example, we may expect to require a larger number of actuators to achieve the same level of shape correction. To investigate this possibility we included all 135 degrees of freedom in the distortion measure by using the unit matrix for the weighting matrix B.

Table 3 shows the distortion ratios for this case. As expected, the number of actuators required to achieve a given value of the distortion ratio increases greatly even for the full measurement case. For

Table 4 Actuator locations for reflector surface z displacements

Location of actuators	Number of actuators, $\rho = 0.99999$								Number of actuators, $\rho = 0.0$							
	2	4	6	8	16	24	32	40	2	4	6	8	16	24	32	40
1						•		•						•	•	•
2															•	•
4						•	•	•					•	•	•	•
5					•			•						•	•	•
6	•							•					•	•	•	•
7							•	•						•		•
8							•	•					•	•	•	•
9						•	•	•					•		•	•
12								•								
13				•	•											•
14					•										•	
15						•										•
16															•	
17						•										
18					•	•										
20							•									
21						•										
22					•											•
23															•	
24								•						•		
26						•										•
28							•								•	•
30				•												
31	•														•	
32						•										
37						•		•								
42								•								•
44															•	
46																•
47						•										
50							•									•
55							•									
56								•								
58																•
61					•		•									
66								•								
67								•								
68								•								
73							•									
77																•
79						•							•			
81							•								•	•
82						•										
83								•								
91													•			
92								•								
93								•							•	
95														•		
97			•	•	•		•	•		•	•	•	•	•	•	•
98			•	•	•	•	•	•		•		•	•	•	•	•
99			•	•	•		•	•	•	•	•	•	•	•	•	•
100		•				•	•	•	•	•	•	•	•	•	•	•
101		•				•	•	•			•	•	•	•		•
102		•			•	•	•	•			•	•	•		•	•
103		•	•	•	•			•				•	•	•	•	•
104			•	•	•	•	•				•		•	•	•	•
105			•	•	•	•	•	•				•	•		•	•
106								•								
107						•	•									
109							•	•								
110							•	•						•	•	
111							•	•							•	•
112								•								
113					•			•							•	
117							•									•
118																•
120																•
121															•	
122															•	•
123																•

continued

Table 4 (Continued) Actuator locations for reflector surface z displacements

Location of actuators	Number of actuators, $\rho = 0.99999$								Number of actuators, $\rho = 0.0$							
	2	4	6	8	16	24	32	40	2	4	6	8	16	24	32	40
124								•						•		
125														•		
126							•									•
127							•	•								
128							•									
129						•	•							•		
130						•									•	
131							•							•		
132																•
133					•		•									
134								•						•		
135					•											
136															•	
138																•
139																•
140														•		
142								•					•		•	
143							•							•		•
144								•								•
146								•							•	
148						•		•								
149						•	•	•						•		

example, with 27 distortion degrees of freedom, eight actuators were sufficient for a distortion ratio of about 0.25 (see Tables 1b and 1c). Now we need more than about 20 actuators for similar performance. As in Table 1, the distortion ratio improves with the reduction in correlation coefficient, but on including the effect of the increase in $E(f)$ the corrected distortion actually increases. The performance with partial measurements is still quite dismal, although, as the performance with complete measurements is not as good, the former does not look as bad in comparison. Still, with 40 pairs of sensors and actuators we can achieve only a distortion ratio of about 0.4.

Table 4 shows the distribution of actuator locations obtained by the genetic algorithm for the fully correlated and uncorrelated cases, respectively. For the completely correlated model, the distribution of the best location of actuators does not appear to be so consistent. For the uncorrelated case the best locations follow a pattern as the number of actuators increases, with the locations used for a low number of actuators retained and neighboring locations added. This pattern does not hold for the fully correlated case.

Concluding Remarks

Expressions for the statistics of shape distortion of space truss structures when measurements are limited to the distortions in a subset of the members of the truss were obtained for both the uncorrected and corrected cases. It was shown that the mean values and covariance matrices of the errors can be used to estimate the distortions in the rest of the truss members. Furthermore, the best strategy for controlling the distortion was shown to be the treatment of estimated member distortions as if they had been measured.

The performance of shape correction was investigated for a 150-member antenna truss. A genetic algorithm was employed for placing pairs of sensors and actuators so as to minimize the expected value of the distortion. Results were obtained for two cases, one with a distortion measure including only axial displacements on the reflector face of the truss, and the other including all degrees of freedom. For the first case it was found that with complete measurement the distortion of the truss could be reduced effectively with a small number of actuators. With partial measurements even a large number of actuators did not provide substantial reduction in shape distortion. For the case when all degrees of freedom were involved, a large number of actuators was required even with complete measurement. The performance was still dismal with partial measurements, but the performance discrepancy due to having only partial measurements was not as great.

Appendix A: Conditional Probabilities

The derivations of the conditional expected value and correlation are summarized from Ref. 5, as follows:

The conditional density function of ϵ_1 is given by fixing the element of ϵ_2 as

$$r(\epsilon_1 \mid \epsilon_2) = \frac{e(\epsilon_1, \epsilon_2)}{h(\epsilon_2)} \tag{A1}$$

where e is the joint density of the complete set of $\epsilon (= [\epsilon_1^T, \epsilon_2^T]^T)$, and h is the joint density of the ϵ_2. Assuming that the ϵ is a normally distributed random variable,

$$h(\epsilon_2) = \frac{1}{(2\pi)^{q/2}|\Sigma_{22}|^{1/2}} \exp\left[-\tfrac{1}{2}(\epsilon_2 - \mu_2)^T \Sigma_{22}^{-1}(\epsilon_2 - \mu_2)\right] \tag{A2}$$

where μ_1 and μ_2 are expected values of ϵ_1 and ϵ_2, respectively. Consider

$$e(\epsilon) = \frac{1}{(2\pi)^{(p+q)/2}|\Sigma|^{1/2}} \exp\left[-\tfrac{1}{2}(\epsilon - \mu)^T \Sigma^{-1}(\epsilon - \mu)\right] \tag{A3}$$

where

$$|\Sigma| = |\Sigma_{22}|\left|\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\right| \tag{A4}$$

and

$$\Sigma^{-1} = \begin{bmatrix} \left(\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\right)^{-1} & -\left(\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\right)^{-1}\Sigma_{12}\Sigma_{22}^{-1} \\ -\Sigma_{22}^{-1}\Sigma_{21}\left(\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\right)^{-1} & \Sigma_{22}^{-1} + \Sigma_{22}^{-1}\Sigma_{21}\left(\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\right)^{-1}\Sigma_{12}\Sigma_{22}^{-1} \end{bmatrix} \tag{A5}$$

Thus, the conditional density function r is obtained as

$$r(\epsilon_1 \mid \epsilon_2) = \frac{1}{(2\pi)^{p/2}|\Sigma_{11m}|^{1/2}} \times \exp\left[-\tfrac{1}{2}(\epsilon_1 - \mu_{1m})^T \Sigma_{11m}^{-1}(\epsilon_1 - \mu_{1m})\right] \tag{A6}$$

where

$$\mu_{1m} = E_m(\epsilon_1) = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(\epsilon_{2m} - \mu_2) \tag{A7}$$

and

$$\Sigma_{11m} = \Sigma_m(\epsilon_1) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T \tag{A8}$$

Appendix B: Expected Value of f_{cm}

The expected value of f_{cm} [Eq. (29)] is given as

$$E(f_{cm}) = E(\epsilon^{pT} A \epsilon^p) - E\Big[\epsilon^{pT} L_s L_s^T E_m(\epsilon^p) + E_m(\epsilon^p)^T L_s L_s^T \epsilon^p - E_m(\epsilon^p)^T L_s L_s^T E_m(\epsilon^p)\Big] \tag{B1}$$

For simplifying Eq. (B1) we need the following relationship for arbitrary x and y:

$$E\big(x^T L_s L_s^T y\big) = E\big[\mathrm{trace}\big(L_s^T y x^T L_s\big)\big] = \mathrm{trace}\big[E\big(L_s^T y x^T L_s\big)\big] = \mathrm{trace}\big[L_s^T E(y x^T) L_s\big] \tag{B2}$$

With the help of Eq. (B2), we transform Eq. (B1) into

$$E(f_{cm}) = E(\epsilon^{pT} A \epsilon^p) - \mathrm{trace}\Big\{L_s^T E\Big[\epsilon^p \epsilon^{pT} - \epsilon^p \epsilon^{pT} + E_m(\epsilon^p)\epsilon^{pT} + \epsilon^p E_m(\epsilon^p) - E_m(\epsilon^p) E_m(\epsilon^p)^T\Big] L_s\Big\} \tag{B3}$$

Finally, the additional application of Eq. (B2) yields

$$E(f_{cm}) = E[\epsilon^{pT}(A - S)\epsilon^p] + E\Big\{\big[\epsilon^p - E_m(\epsilon^p)\big]^T S \big[\epsilon^p - E_m(\epsilon^p)\big]\Big\} \tag{B4}$$

The second term of the right-hand side of Eq. (B4) is

$$\begin{aligned} &E\Big\{\big[\epsilon^p - E_m(\epsilon^p)\big]^T S \big[\epsilon^p - E_m(\epsilon^p)\big]\Big\} \\ &= E\Big[\mathrm{trace}\Big\{L_s^T\big[\epsilon^p - E_m(\epsilon^p)\big]\big[\epsilon - E_m(\epsilon^p)\big]^T L_s^T\Big\}\Big] \\ &= \mathrm{trace}\Big\{L_s^T E\Big[\big(\epsilon_1^T - \mu_{1m}^T, 0^T\big)^T\big(\epsilon_1^T - \mu_{1m}^T, 0^T\big)\Big] L_s\Big\} \\ &= \mathrm{trace}\left(L_s^T \begin{bmatrix} \Sigma_{11m} & 0 \\ 0 & 0 \end{bmatrix} L_s\right) \geq 0 \end{aligned} \tag{B5}$$

Also, clearly, $E[\epsilon^{pT}(A - S)\epsilon^p] = E[\epsilon^T(A - S)\epsilon]$, and so, using Eq. (23), we have

$$E(f_{cm}) = E(f_c) + \mathrm{trace}\left(L_s^T \begin{bmatrix} \Sigma_{11m} & 0 \\ 0 & 0 \end{bmatrix} L_s\right) \tag{B6}$$

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E. A. Thornton
Associate Editor